12-1 Adding and Subtracting Matrices

Vocabulary

Review

1. Circle the words whose meaning is similar to that of *corresponding*.

different (equivalent) intersecting (matching) (related) (similar)

Vocabulary Builder

matrix (noun) MAY triks

Related Word: matrices (plural)

Definition: A **matrix** is a rectangular array of numbers written within brackets. A **matrix** with m horizontal rows and n vertical columns is an $m \times n$ **matrix**.

a 2 × 3 **matrix**

$$\begin{bmatrix} -5 & 3 & 1 \\ 7 & 12 & -4 \end{bmatrix}$$

Use Your Vocabulary

Write T for *true* or F for *false*.

T 2. The *matrix* $\begin{bmatrix} 4 & -2 \\ 0 & 7 \end{bmatrix}$ has two horizontal rows and two vertical columns.

T 3. The *matrix* $\begin{bmatrix} 25 & 3 \\ -2 & -18 \\ 4 & 13 \end{bmatrix}$ is a 3×2 *matrix*.

4. Write an example of a 4×3 *matrix*.

Sample: $\begin{bmatrix} 4 & 2 & 0 \\ 8 & 3 & -7 \\ 1 & 1 & 0 \\ 4 & 6 & -9 \end{bmatrix}$

5. Write an example of a *matrix* with one column.

Sample: $\begin{bmatrix} 5 \\ -11 \\ 0 \end{bmatrix}$

Take note

Key Concept Matrix Addition and Subtraction

To add matrices A and B with the same dimensions, add *corresponding* elements. Similarly, to subtract matrices A and B with the same dimensions, subtract *corresponding* elements. *Corresponding* elements are elements in the same position in each matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix} \qquad A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$$

Problem 1 Adding and Subtracting Matrices

Got It? Given $A = \begin{bmatrix} -12 & 24 \\ -3 & 5 \\ -1 & 10 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 2 & -4 \\ -1 & 5 \end{bmatrix}$, what is $A + B$?

6. Use the justifications at the right to add the matrices.

$$A + B = \begin{bmatrix} -12 & 24 \\ -3 & 5 \\ -1 & 10 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 2 & -4 \\ -1 & 5 \end{bmatrix}$$ Write the original matrices.

$$= \begin{bmatrix} -12 + (-3) & 24 + 1 \\ -3 + 2 & 5 + (-4) \\ -1 + (-1) & 10 + 5 \end{bmatrix}$$ Add corresponding elements.

$$= \begin{bmatrix} -15 & 25 \\ -1 & 1 \\ -2 & 15 \end{bmatrix}$$ Simplify.

Problem 2 Solving a Matrix Equation

Got It? If $B = \begin{bmatrix} 1 & 6 & -1 \\ 2 & 6 & 1 \\ -1 & -2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 & 0 \\ -1 & -3 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, and $A - B = C$, what is A?

7. To solve the equation $A - B = C$ for A, you add B to / subtract B from both sides of the equation.

8. Solve the equation $A - B = C$ for A. Write your answer below.

$A = C + B$

9. Use your answer from Exercise 8 and the values of B and C to find matrix A.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & -3 & 6 \\ 2 & 3 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 6 & -1 \\ 2 & 6 & 1 \\ -1 & -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & -1 \\ 1 & 3 & 7 \\ 2 & 1 & -3 \end{bmatrix}$$

For $m \times n$ matrices, the additive identity matrix is the zero matrix, O, with all elements zero. The *opposite*, or *additive inverse*, of an $m \times n$ matrix A is $-A$, where each element is the opposite of the corresponding element of A.

Problem 3 Using Identity and Opposite Matrices

Got It? What is the sum $\begin{bmatrix} 14 & 5 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -14 & -5 \\ 0 & 2 \end{bmatrix}$?

10. **Multiple Choice** Which matrix is equal to $\begin{bmatrix} 14 & 5 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -14 & -5 \\ 0 & 2 \end{bmatrix}$?

(A) $\begin{bmatrix} 14 & 5 \\ 0 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} 28 & 10 \\ 0 & -4 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Got It? What is the sum $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 10 & -5 \\ 0 & 2 & -3 \end{bmatrix}$?

11. The matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is the additive inverse / zero matrix.

12. The sum of the two matrices is $\begin{bmatrix} -1 & 10 & -5 \\ 0 & 2 & -3 \end{bmatrix}$.

Problem 4 Finding Unknown Matrix Values

Got It? What values of x and y make the following equation true?

$$\begin{bmatrix} x+3 & -2 \\ y-1 & x+1 \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ 2y+5 & 7 \end{bmatrix}$$

13. Equal matrices have the same / different dimensions and equal / unequal corresponding elements.

14. Explain how you can solve for the values of x and y.

Answers may vary. Sample: Set the corresponding elements equal and solve the equations that have variables.

15. Complete the steps to solve for y.

$2y + 5 = y - 1$	Corresponding elements are equal.
$2y - y = -1 - 5$	Group like terms on the same side.
$y = -6$	Simplify.

16. Solve for x.

$x + 3 = 9$ or $x + 1 = 7$

$x = 9 - 3$ $x = 7 - 1$

$x = 6$ $x = 6$

17. The values of x and y that make the equation true are $x =$ 6 and $y =$ −6.

Lesson Check • Do you UNDERSTAND?

Vocabulary Are the two matrices equal? Explain.

$$\begin{bmatrix} \frac{1}{2} & \frac{3}{8} \\ 0.2 & \sqrt[3]{27} \end{bmatrix} \text{ and } \begin{bmatrix} 0.5 & 0.375 \\ \frac{1}{5} & 3 \end{bmatrix}$$

18. Circle the items that you must check to be sure the two matrices are equal.

(corresponding elements) additive inverses (number of columns) (number of rows)

19. Are the two matrices equal? Explain how you know.

Yes. Answers may vary. Sample: The dimensions of the matrices are equal. Set each pair of corresponding elements equal to each other and verify. $\frac{1}{2} = 0.5$, $\frac{3}{8} = 0.375$, $0.2 = \frac{1}{5}$, and $\sqrt[3]{27} = 3$.

Math Success

Check off the vocabulary words that you understand.

☐ matrix ☐ corresponding elements ☐ matrix equation ☐ zero matrix

Rate how well you can *add and subtract matrices and solve matrix equations.*

Need to review 0 2 4 6 8 10 Now I get it!