

10-5

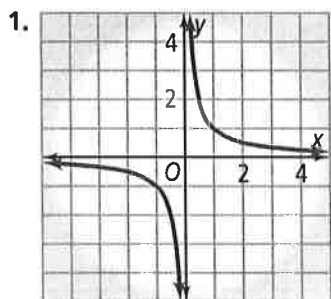
Hyperbolas



Vocabulary

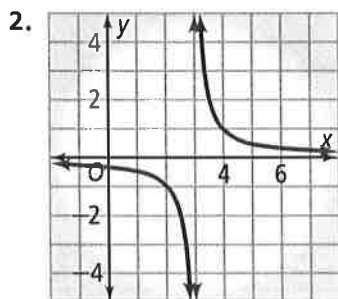
Review

Write the equation of the vertical and horizontal *asymptotes* in each graph.



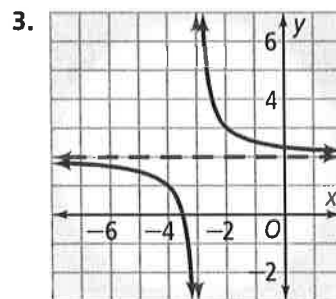
Vertical: $x = 0$

Horizontal: $y = 0$



Vertical: $x = 3$

Horizontal: $y = 0$



Vertical: $x = -3$

Horizontal: $y = 2$

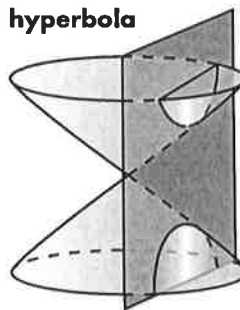
Vocabulary Builder

hyperbola (noun) hy PUR buh luh

Definition: When a plane, parallel to the axis of a *double cone*, slices the double cone, a *conic section* called a **hyperbola** is formed.

Main Idea: A hyperbola has two foci, two vertices, and two curves.

hyperbola



Use Your Vocabulary

Write T for *true* or F for *false*.

F 4. The graph of a hyperbola consists of only one curve.

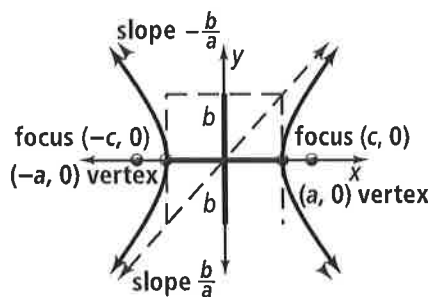
F 5. A hyperbola is *not* a conic section.

T 6. A hyperbola has two vertices and two foci.

Take note

Key Concept Properties of Hyperbolas with Centers (0, 0)

Horizontal Hyperbola



Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

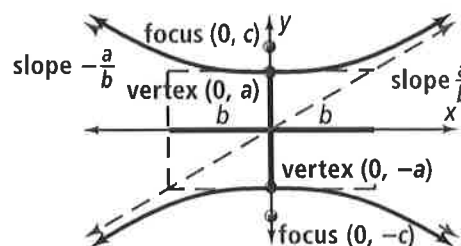
Transverse axis: Horizontal

Vertices $(\pm a, 0)$

Foci: $(\pm c, 0)$, where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm \frac{b}{a}x$

Vertical Hyperbola



Equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Transverse axis: Vertical

Vertices $(0, \pm a)$

Foci: $(0, \pm c)$, where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm \frac{a}{b}x$



Problem 1 Writing and Graphing the Equation of a Hyperbola

Got It? What is the standard-form equation of the hyperbola with the vertices $(0, \pm 4)$ and foci $(0, \pm 5)$?

7. The vertices are $(0, \pm 4)$, so $a = 4$.
8. The foci are $(0, \pm 5)$. Then $c = 5$.
9. Use the values you found for a and c and $c^2 = a^2 + b^2$ to find b .

$$\begin{aligned} 5^2 &= 4^2 + b^2 \\ 25 &= 16 + b^2 \\ 9 &= b^2 \\ 3 &= b \end{aligned}$$

10. The vertices and foci are on the x -axis / y -axis.
11. The transverse axis of the hyperbola is horizontal / vertical.
12. Complete the steps to find the standard form of the equation of the hyperbola.

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	Write the standard form.
$\frac{y^2}{4} - \frac{x^2}{3} = 1$	Substitute values for a and b .
$\frac{y^2}{16} - \frac{x^2}{9} = 1$	Simplify.



Problem 2 Analyzing a Hyperbola From Its Equation

Got It? What are the vertices, foci, and asymptotes of the hyperbola with equation $9x^2 - 4y^2 = 36$? Sketch a graph. Use a graphing calculator to check your sketch.

13. Write the equation in standard form. Divide each side by the same number to get 1 on the right side.

$$9x^2 - 4y^2 = 36$$

$$\frac{9x^2}{36} - \frac{4y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

14. Circle all statements that are *true* for this hyperbola.

The transverse axis is horizontal.

The transverse axis is vertical.

The vertices are on the y -axis.

The vertices are on the x -axis.

The foci are on the y -axis.

The foci are on the x -axis.

15. Use the standard form of the equation to identify the values of a^2 , a , b^2 , and b .

$$a^2 = 4 \quad a = 2 \quad b^2 = 9 \quad b = 3$$

16. Use the values you found for a and b and $c^2 = a^2 + b^2$ to find c^2 and c .

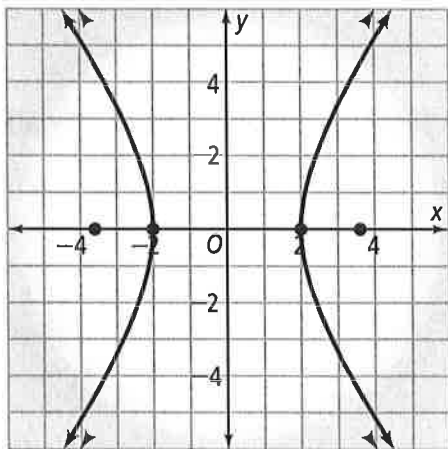
$$c^2 = 13 \quad c = \sqrt{13}$$

17. The vertices are $(\pm 2, 0)$ and the foci are $(\pm \sqrt{13}, 0)$.

The slopes of the asymptotes are $m = \pm \frac{b}{a}$, so $m = \pm \frac{3}{2}$.

The equations of the asymptote are $y = \pm \frac{3}{2}x$.

18. Plot the vertices and foci. Draw dashed lines for the asymptotes. Then sketch a graph the hyperbola.



19. Solve the equation for y .

$$\frac{x^2}{4} - \frac{y^2}{9} = 1 \quad y^2 = 9\left(\frac{x^2}{4} - 1\right)$$

$$\frac{y^2}{9} = \frac{x^2}{4} - 1 \quad y = 3\sqrt{\frac{x^2}{4} - 1} \text{ and } y = -3\sqrt{\frac{x^2}{4} - 1}$$

20. Use your graphing calculator to graph the two equations you found in Exercise 19 on the same screen. Compare the graph on your screen to your sketch.

Answers may vary. Sample: The graph on my calculator screen looks

just like my sketch.



Lesson Check • Do you UNDERSTAND?

Error Analysis Your friend says that a graph must be a vertical hyperbola because the greater denominator is under the y^2 term. What error did your friend make?

21. Circle the equation that represents a vertical hyperbola. Underline the equation that represents a horizontal hyperbola

$$\frac{y^2}{36} - \frac{x^2}{9} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{36} = 1$$

22. Name one way the two equations differ.

Answers may vary. Sample: The equation for the vertical hyperbola has a

positive y^2 term, while the equation for the horizontal hyperbola has a

negative y^2 term.



Math Success

Check off the vocabulary words that you understand.

- hyperbola focus of the hyperbola vertex transverse axis

Rate how well you can graph a hyperbola.

