8-1 Inverse Variation

Vocabulary

Review

1. Cross out the expressions that do NOT contain a *constant* term.

$6y + 5$ $\quad x^2 + 3y$ $\quad 7y + \frac{-6}{x} + 2$ $\quad x - 7y + z$

2. In the expression $7x^2 + 3x + 1$, what is the value of the *constant*? Circle your answer.

1 $\quad$ 2 $\quad$ 3 $\quad$ 7

Vocabulary Builder

inverse variation (noun) IN vurs vehr ee AY shun

Definition: An **inverse variation** is a relationship between two quantities where one quantity increases as the other decreases by the same factor, k.

inverse variation

$y = \frac{k}{x}$, $xy = k$, or $x = \frac{k}{y}$

where $k \neq 0$

Main Idea: Two quantities vary *inversely* when one quantity increases as the other decreases proportionally.

Example: The time to complete a race decreases as average speed increases. This relationship between time and speed is an **inverse variation.**

Nonexample: As the force with which you throw a ball increases, the distance it travels also increases. The relationship between force and distance a ball is thrown is a *direct variation*, not an **inverse variation.**

Use Your Vocabulary

Write T for *true* or F for *false*.

___ 3. As the radius of a pizza increases, the circumference of the pizza also increases. This relationship represents a *direct variation*.

___ 4. As the number of miles a car is driven increases, the number of gallons of gas in the car's tank decreases. This relationship represents an *inverse variation*.

___ 5. As the number of pages in a book increases, the weight of the book increases. This relationship represents an *inverse variation*.

In an *inverse variation*, the product of every x-value with its corresponding y-value is the same number. In a *direct variation*, the ratio of every y-value to its corresponding x-value is the same number.

Problem 1 Identifying Direct and Inverse Variations

Got It? **Is the relationship between the variables a *direct variation*, an *inverse variation*, or *neither*? Write function models for the direct and inverse variations.**

x	y
0.2	8
0.5	20
1.0	40
1.5	60

6. Find the value of each expression.

xy	$\frac{y}{x}$
1.6	

7. Underline the correct word or words to complete each sentence.

The product of every x-value and its corresponding y-value is / is not constant.

The ratio of every y-value to its corresponding x-value is / is not constant.

The relationship between the variables represents a(n) direct / inverse variation.

8. Write a function to model the data.

$\frac{y}{x} =$ or $y =$ $\cdot x$

Problem 2 Determining an Inverse Variation

Got It? **Suppose x and y vary inversely, and $x = 8$ when $y = -7$. What is the function that models the inverse variation?**

9. Circle the equation that represents the general form for inverse variation.

$\frac{y}{x} = k$ $xy = k$ $\frac{y}{k} = x$

10. Use the justifications at the right to determine the function.

$xy = k$	Write the general function form for inverse variation.
	Substitute for x and y.
	Solve for k.

11. The function models the inverse variation.

When one quantity varies with respect to two or more quantities, you have *combined variation.* When one quantity varies directly with two or more quantities, you have *joint variation.*

take note

Key Concepts Combined Variations

Combined Variation	Equation Form
z varies jointly with x and y.	$z = kxy$
z varies jointly with x and y and inversely with w.	$z = \frac{kxy}{w}$
z varies jointly with x and inversely with the product wy.	$z = \frac{kx}{wy}$

Identify the combined variation in each equation.

12. $p = ktn$

p varies jointly with t and .

13. $m = \frac{ky}{xn}$

m varies jointly with and inversely with the product .

14. $v = \frac{krm}{z}$

v varies jointly with and and inversely with .

Problem 4 Using Combined Variation

Got It? **The number of bags of mulch you need to mulch a planting area varies jointly with the area to be mulched a in square feet and the depth of the mulch d in feet. If you need 10 bags to mulch 120 ft^2 to a depth of 3 in., how many bags do you need to mulch 200 ft^2 to a depth of 4 in.?**

15. If b is the number of bags of mulch you need, the function that represents the joint variation is $b = k \cdot \quad \cdot \quad .$

16. Substitute the values you are given for a, b, and d into the function. Then solve for k.

17. Use the value of k to write a formula to find the number of bags of mulch you need given the area and depth.

18. Use the equation you wrote in Exercise 17 to find the number of bags for an area of 200 ft^2 to a depth of 4 in.

19. You need bags to mulch 200 ft^2 to a depth of 4 in.

Problem 5 Applying Combined Variation

Got It? **How much potential energy would a 41-kg diver have standing on a 10-m diving platform?**

20. Write the formula for potential energy.

21. Use the given data to find *PE*.

$g = 9.8$ $m =$ $h =$

$PE = gmh$ $PE = 9.8 \cdot \quad \cdot$ $PE =$

The diver has joules of potential energy.

Got It? **An 80-kg diver stands on a 6-m diving platform. At what height should a 40-kg diver stand to have equal potential energy? Do you need to find the potential energy of either diver to solve this? Explain.**

22. Write the formula for potential energy for each diver.

Diver 1: $PE = gmh = 9.8 \cdot \quad \cdot$ Diver 2: $PE = gmh = 9.8 \cdot \quad \cdot$

23. The potential energy of the divers needs to be equal. Substitute the value of PE for Diver 1 into the formula for Diver 2.

$9.8 \cdot \quad \cdot = 9.8 \cdot \quad \cdot h$

24. Solve for the height, *h*.

$$\frac{9.8 \cdot \quad \cdot}{9.8 \cdot} \qquad h =$$

The 40-kg diver should stand at m.

25. Did you need to find the potential energy of either diver? yes no

Lesson Check • Do you UNDERSTAND?

Writing **Describe how the variables in the equation $p = \frac{kqrt}{s}$ are related.**

26. Circle the variable(s) that vary directly with *p*. Draw a box around the variable(s) that vary inversely with *p*.

q r s t

27. Describe how the variables in the given equation are related.

_____

Math Success

Check off the vocabulary words that you understand.

☐ inverse variation ☐ constant of variation ☐ combined variation ☐ joint variation

Rate how well you can *use direct, inverse, and joint variations.*